The Power of Informative Hypotheses

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for a powerful evaluation
of H_i and H_c using BF_{ic}?

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Informative Hypotheses

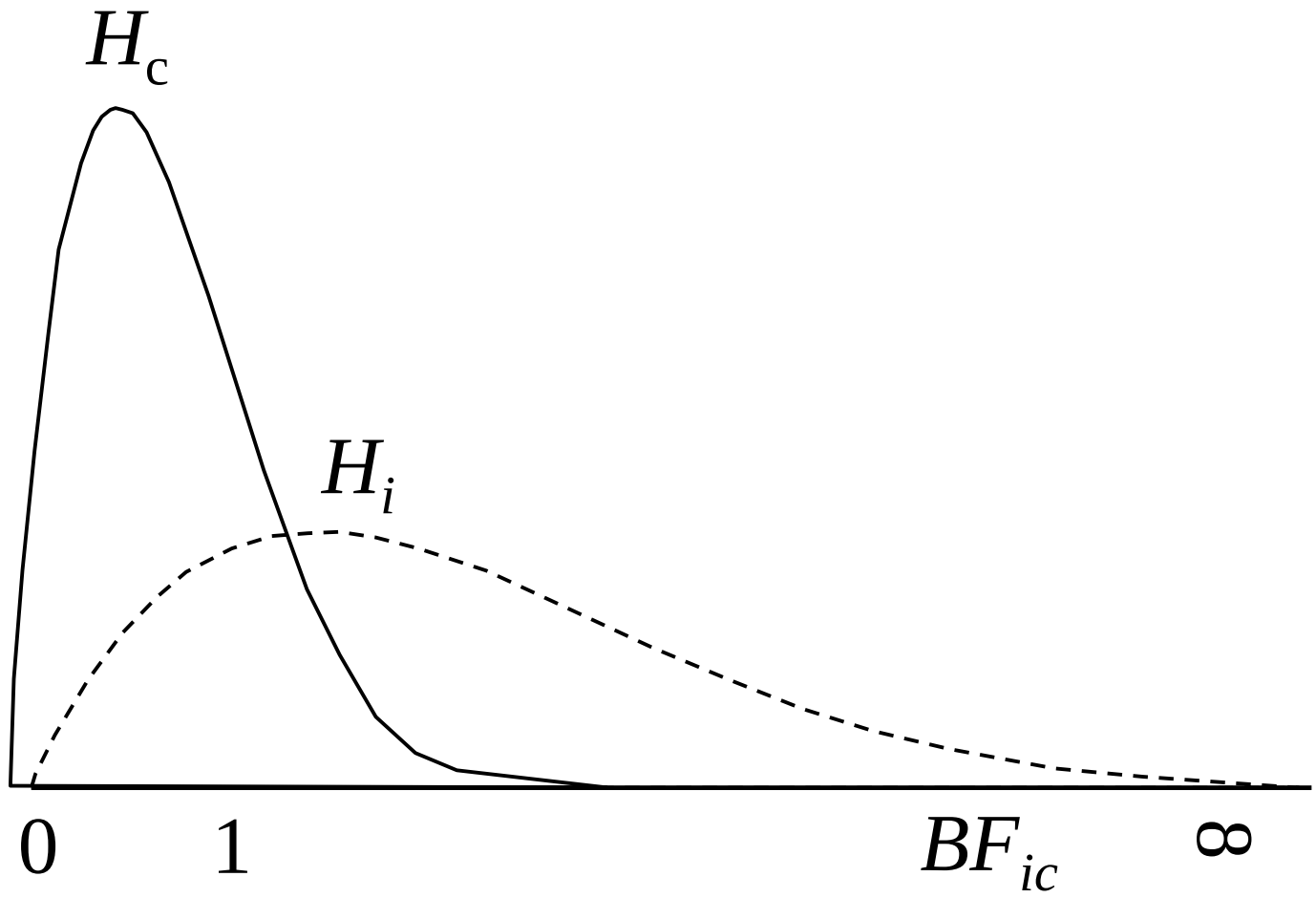
$H_i : \mu_1 > \mu_2 > \mu_3$ $\qquad$ H_c : not H_i

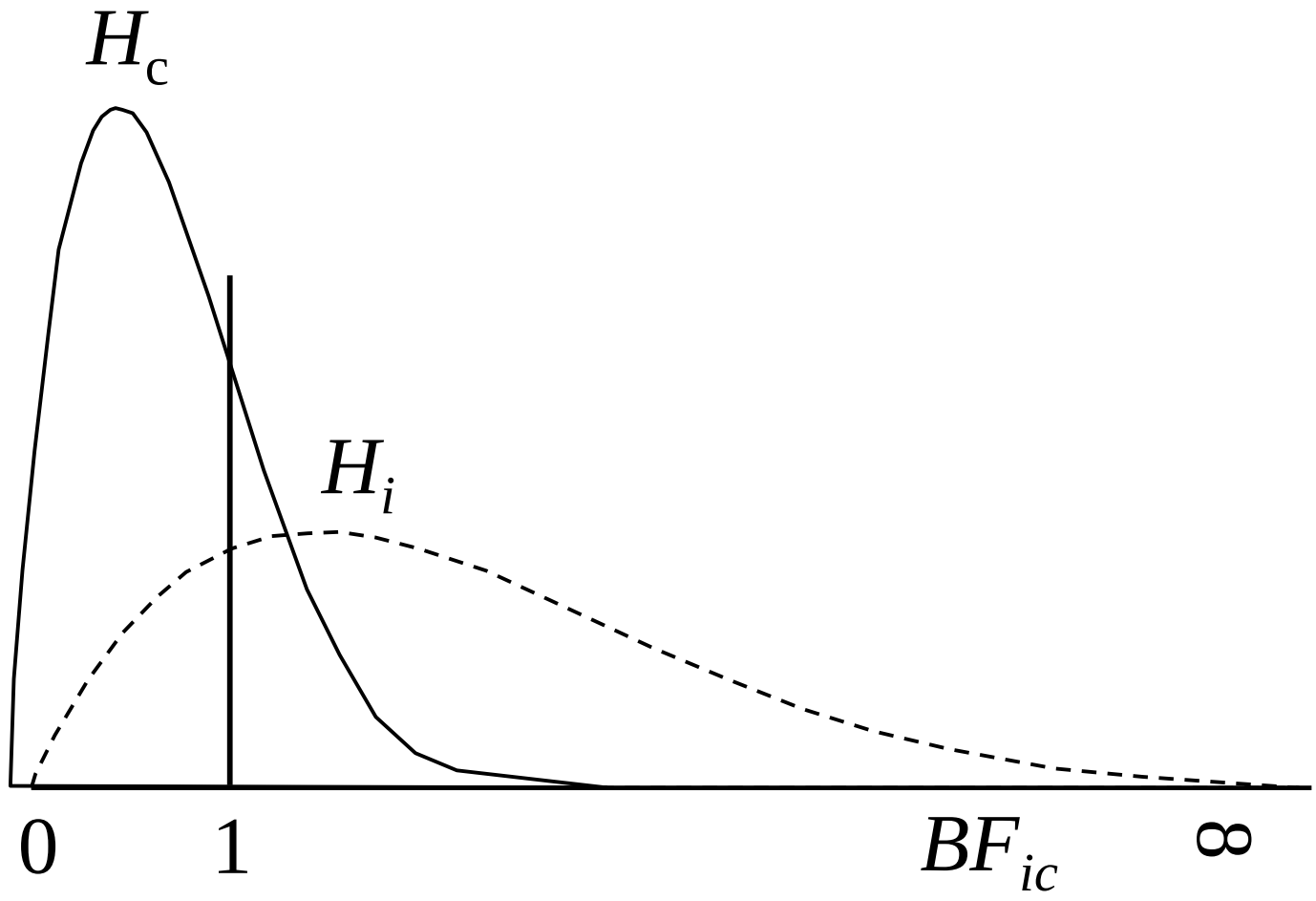
Informative Hypotheses

$H_i : \mu_1 > \mu_2 > \mu_3$ $H_c :$ not H_i BF_{ic}

Power

What is the required sample size
for a powerful evaluation
of H_i and H_c using BF_{ic}?


H_c
H_i
0
1
BF_{ic}
∞


H_c
H_i
0
1
BF_{ic}
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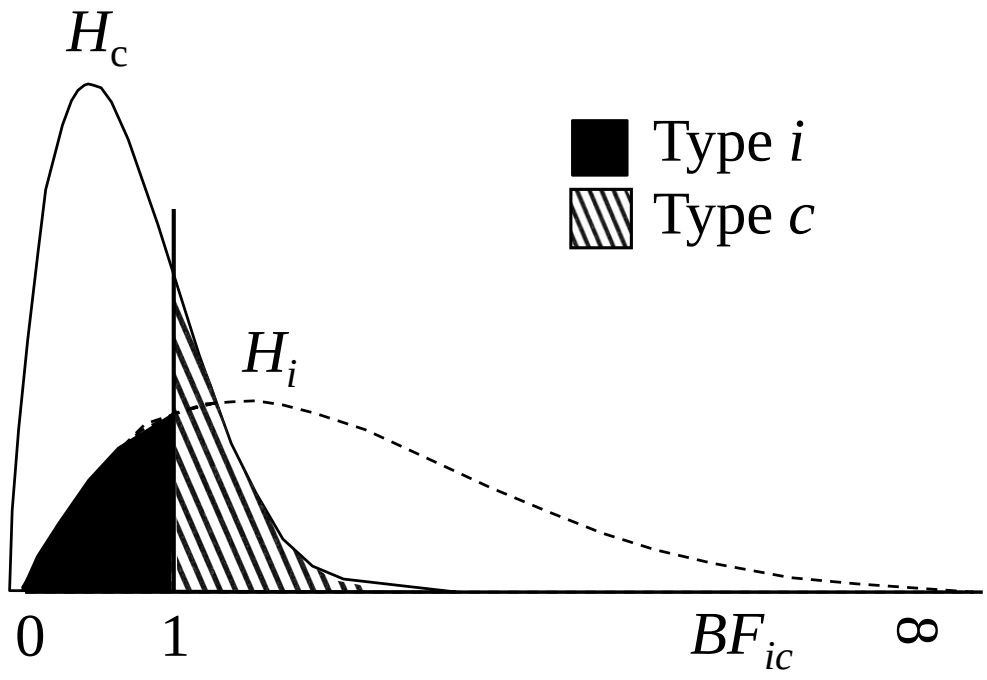
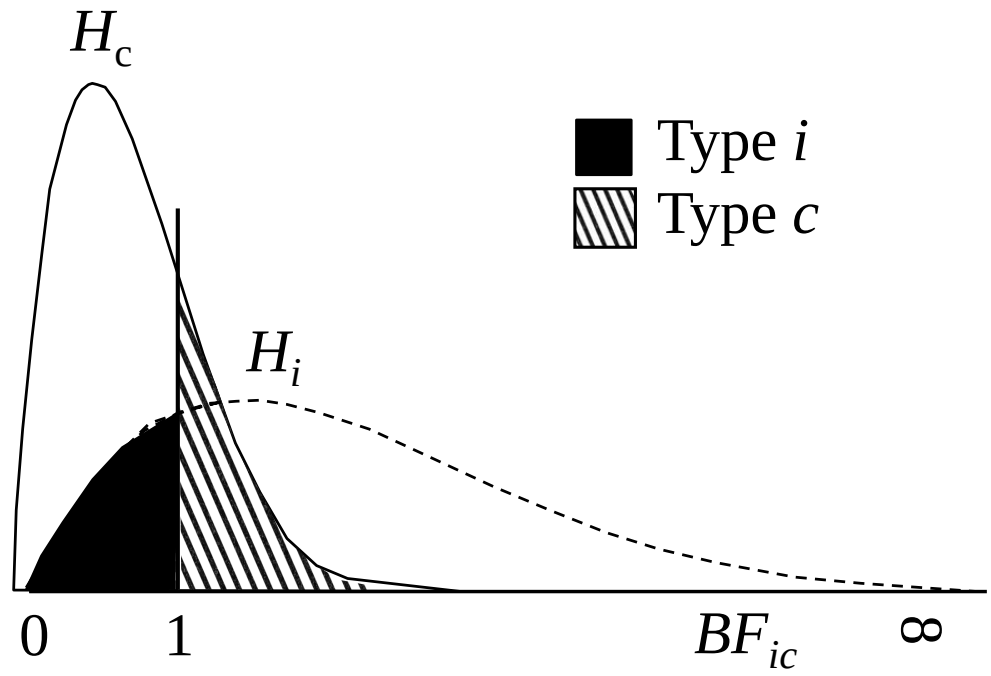
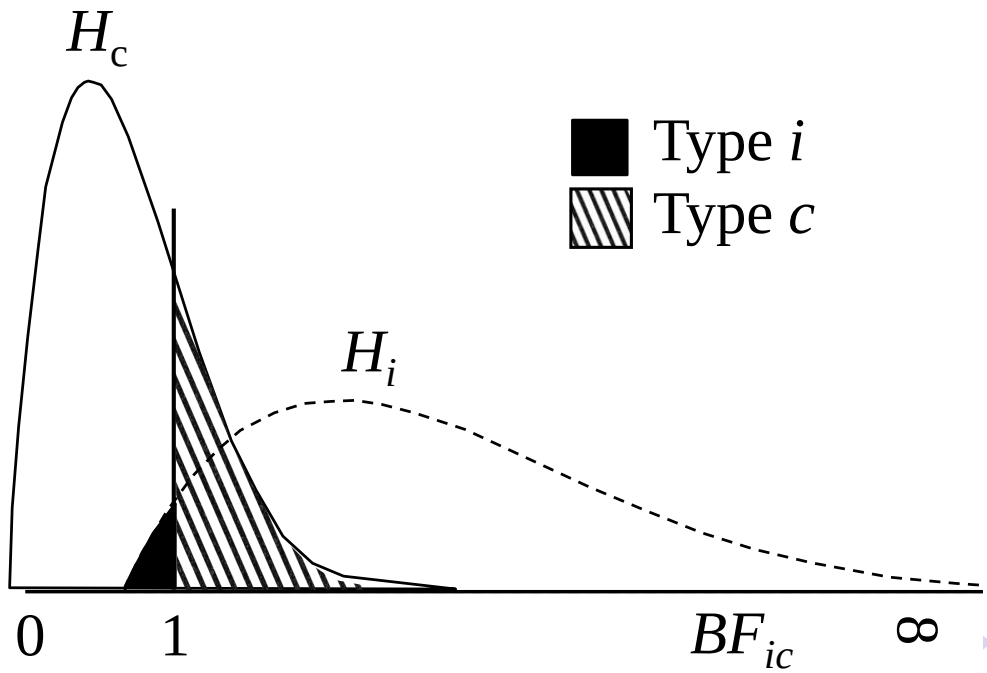
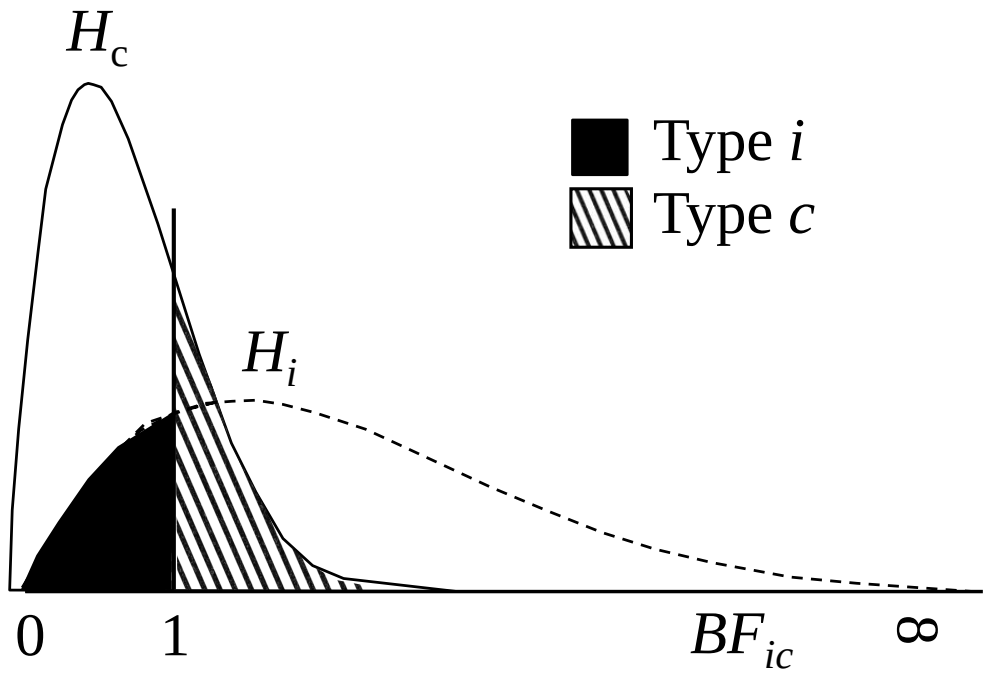
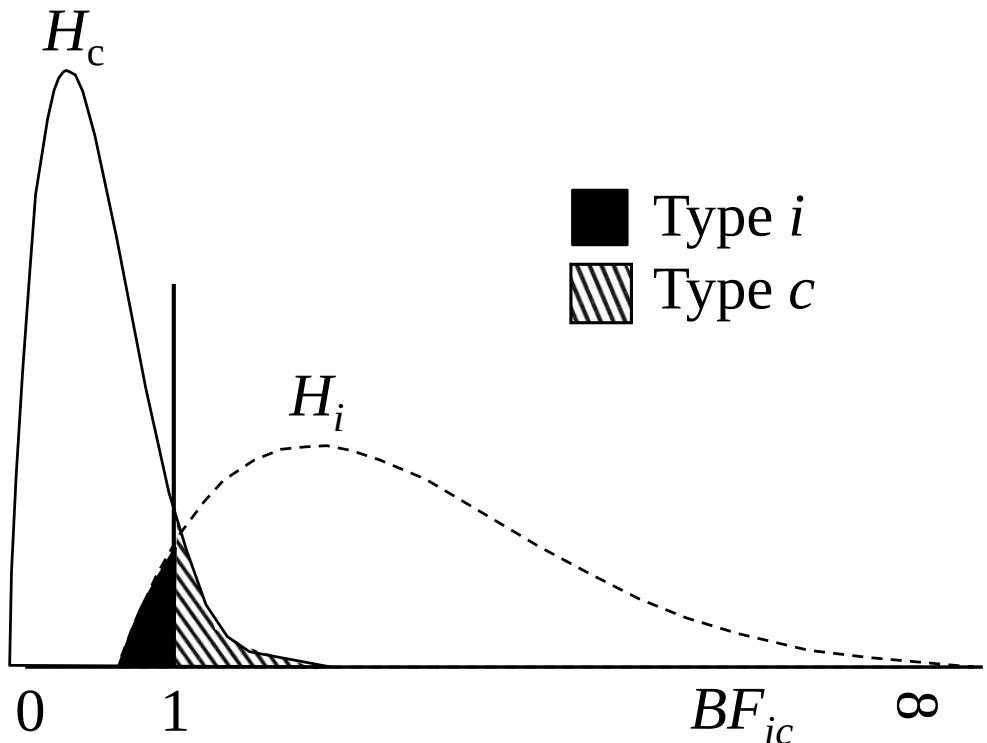
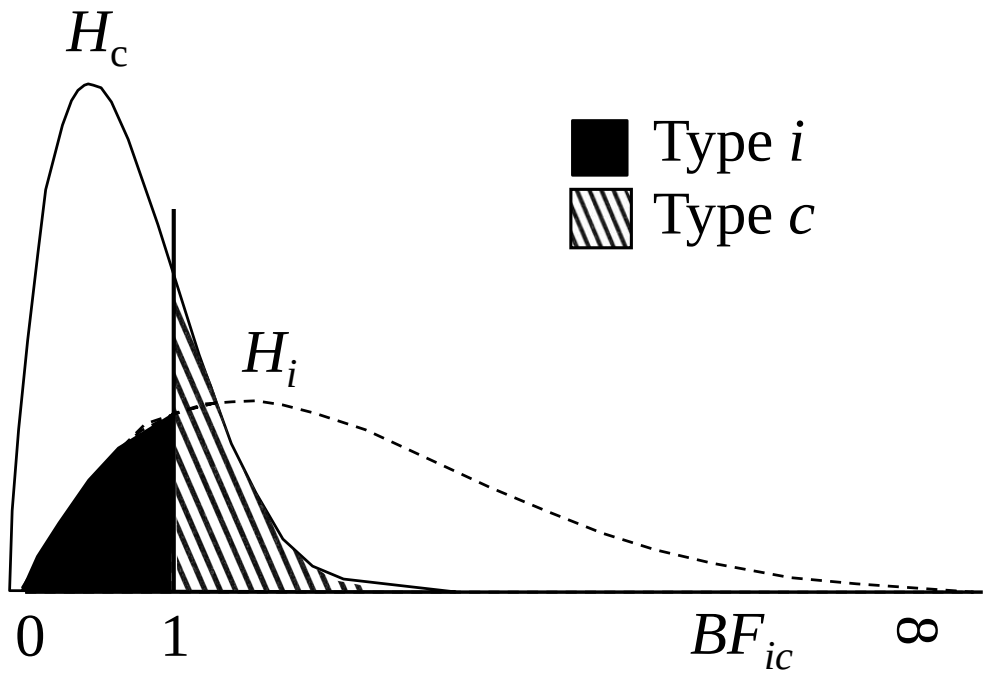

H_c
Type i
Type c
H_i
0
1
BF_{ic}
∞

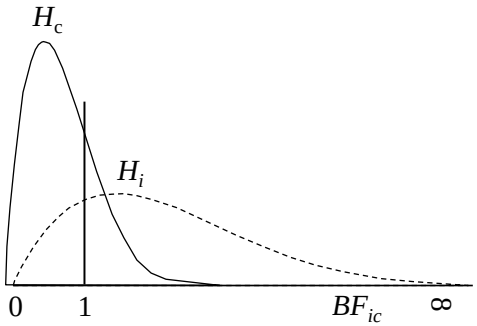

H_c
Type i
Type c
H_i
0
1
BF_{ic}
∞

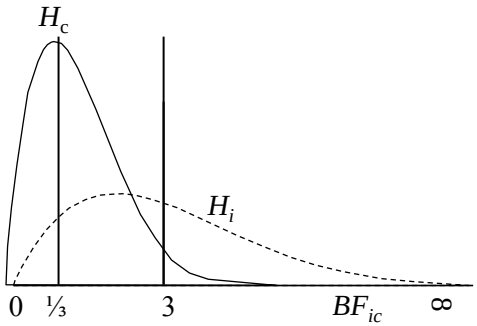

H_c
Type i
Type c
H_i
0
1
BF_{ic}
∞

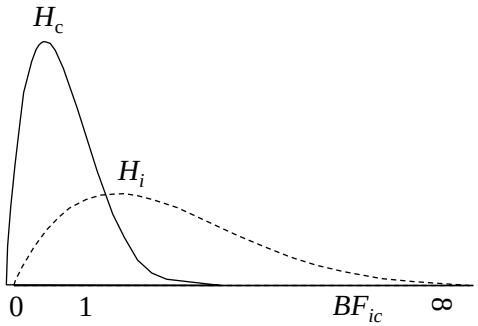

H_c
Type i
Type c
H_i
0
1
BF_{ic}
∞

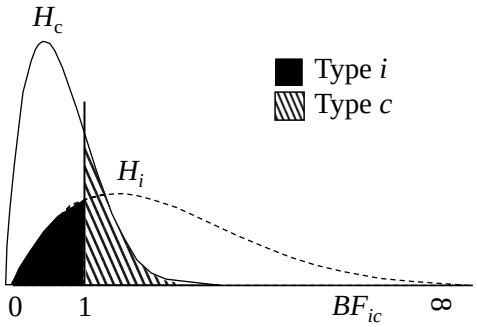

H_c
Type i
Type c
H_i
0
1
BF_{ic}
∞

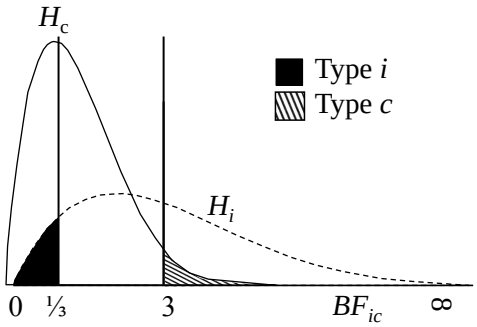

H_c
Type i
Type c
H_i
0
1
BF_{ic}
∞

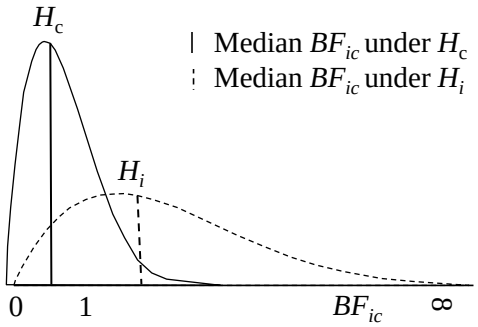

H_c
H_i
0
1
BF_{ic}
∞


H_c
H_i
0
⅓
3
BF_{ic}
∞

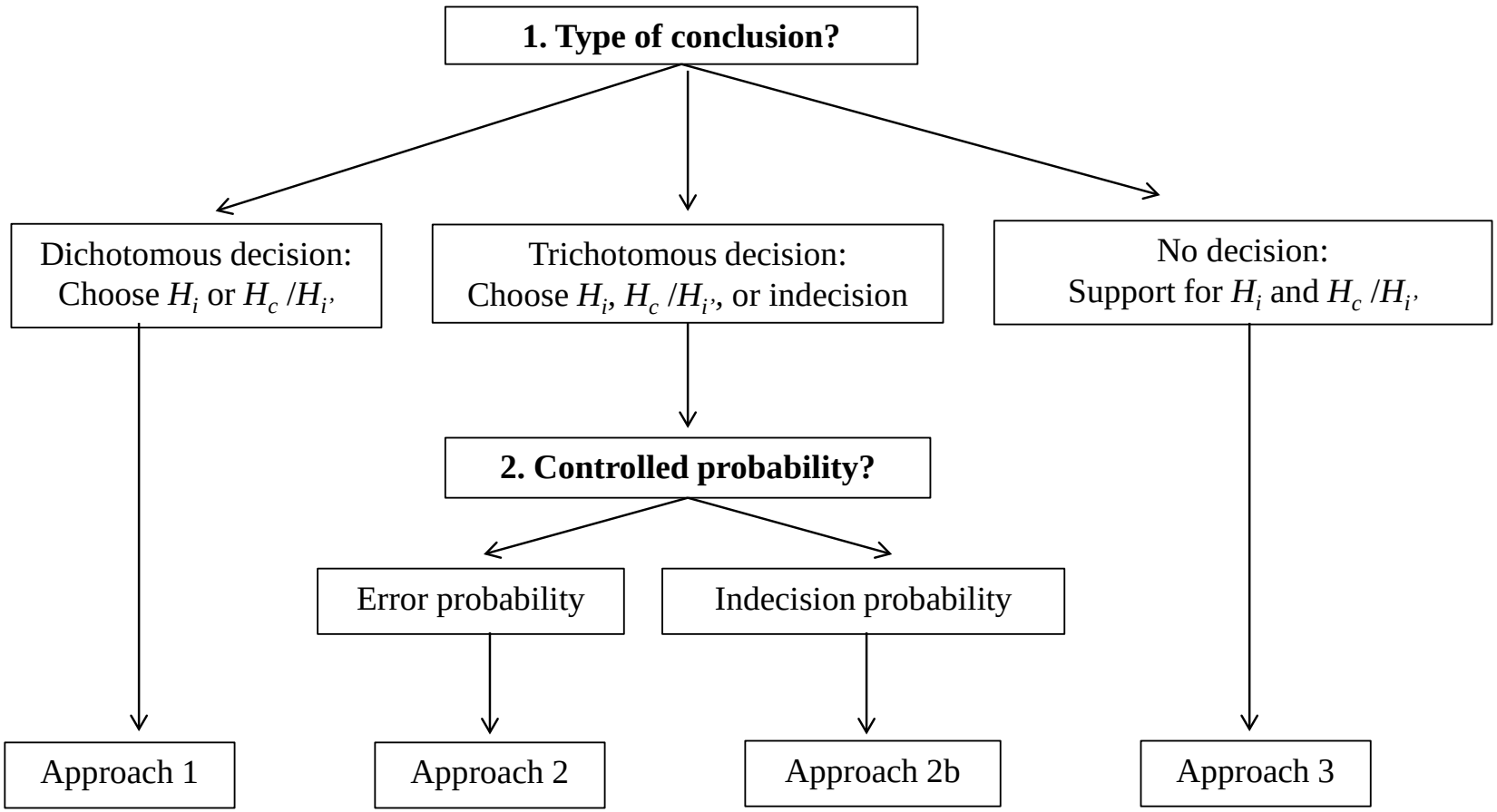

H_c
H_i
0
1
BF_{ic}
∞


H_c
Type i
Type c
H_i
0
1
BF_{ic}
∞


H_c
Type i
Type c
H_i
0
⅓
3
BF_{ic}
∞


H_c
Median BF_{ic} under H_c
Median BF_{ic} under H_i
H_i
0
1
BF_{ic}
∞

Decision tree



Sample size

What is the required sample size
for a powerful evaluation
of H_i and H_c using BF_{ic}?

Simulation procedure

1. Specify *K*,

H_i, and H_c

$K = 3$

H_i: $\mu_1 > \mu_2 > \mu_3$ | H_c: not H_i

2. Specify effect sizes

$d_{H_i} = .2$ | $d_{H_c} = .2$

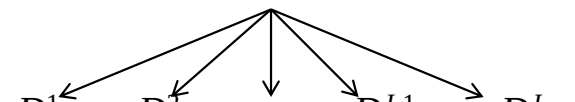
3. Specify population means

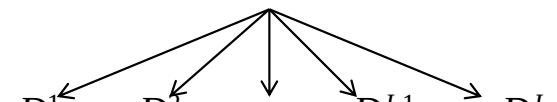
$\mu_1 = .2, \mu_2 = .1, \mu_3 = 0$ | $\mu_1 = .1, \mu_2 = .2, \mu_3 = 0$

4. Specify group sample size *N*

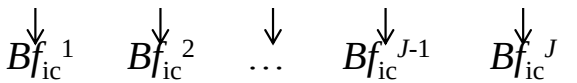
$N = 2$

5. Sample *J* datasets





6. Compute BF_{ic}



7. Compute error probabilities and median Bayes factor for Approach 1-3

8. Increase sample size by 1 in step 4, and repeat step 5-8, until *N* = 1,000. Repeat steps 3-8 for every ordering under H_c

Order of means under H_c

$H_i : \mu_1 > \mu_2 > \mu_3$
Ordering
$\mu_1 > \mu_3 > \mu_2$
$\mu_2 > \mu_1 > \mu_3$
$\mu_2 > \mu_3 > \mu_1$
$\mu_3 > \mu_1 > \mu_2$
$\mu_3 > \mu_2 > \mu_1$

Simulation procedure

1. Specify *K*,

H_i, and H_c

$K = 3$

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2. Specify effect sizes

$d_{H_i} = .2$ | $d_{H_c} = .2$

3. Specify population means

$\mu_1 = .2, \mu_2 = .1, \mu_3 = 0$ | $\mu_1 = .1, \mu_2 = .2, \mu_3 = 0$

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$N = 2$

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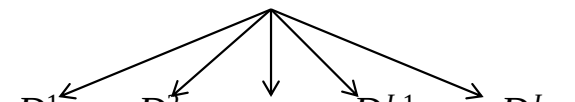
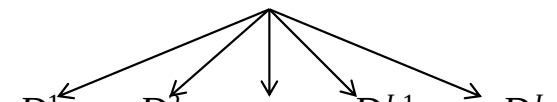
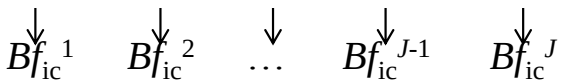




6. Compute BF_{ic}



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8. Increase sample size by 1 in step 4, and repeat step 5-8, until *N* = 1,000. Repeat steps 3-8 for every ordering under H_c

Order of means under H_c

$H_i : \mu_1 > \mu_2 > \mu_3$	
Ordering	Violation of H_i
$\mu_1 > \mu_3 > \mu_2$	small
$\mu_2 > \mu_1 > \mu_3$	small
$\mu_2 > \mu_3 > \mu_1$	medium
$\mu_3 > \mu_1 > \mu_2$	medium
$\mu_3 > \mu_2 > \mu_1$	large

Results Approach 1: H_i vs H_c

	Error probability								
	.1			.05			.025		
d_{H_i}	.2	.5	-	.2	.5	-	.2	.5	-
s	644	624	977	994	977	*	*	*	*
m	136	56	103	222	110	180	313	180	255
l	109	35	54	181	65	100	278	108	148
-	159	28		258	42		361	58	

indicates controlled Type i error probability

indicates controlled Type c error probability

indicates controlled Decision error probability

Results Approach 1: H_i vs H_c

	Error probability								
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d_{H_i}	.2	.5	-	.2	.5	-	.2	.5	-
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-	159	28		258	42		361	58	

indicates controlled Type *i* error probability

indicates controlled Type *c* error probability

indicates controlled Decision error probability

Cohen: $\beta = .2, \alpha = .05$

*f**	.1	.25
ss	322	52

*An effect size *d* of .2 = .007 in *f*

Summary

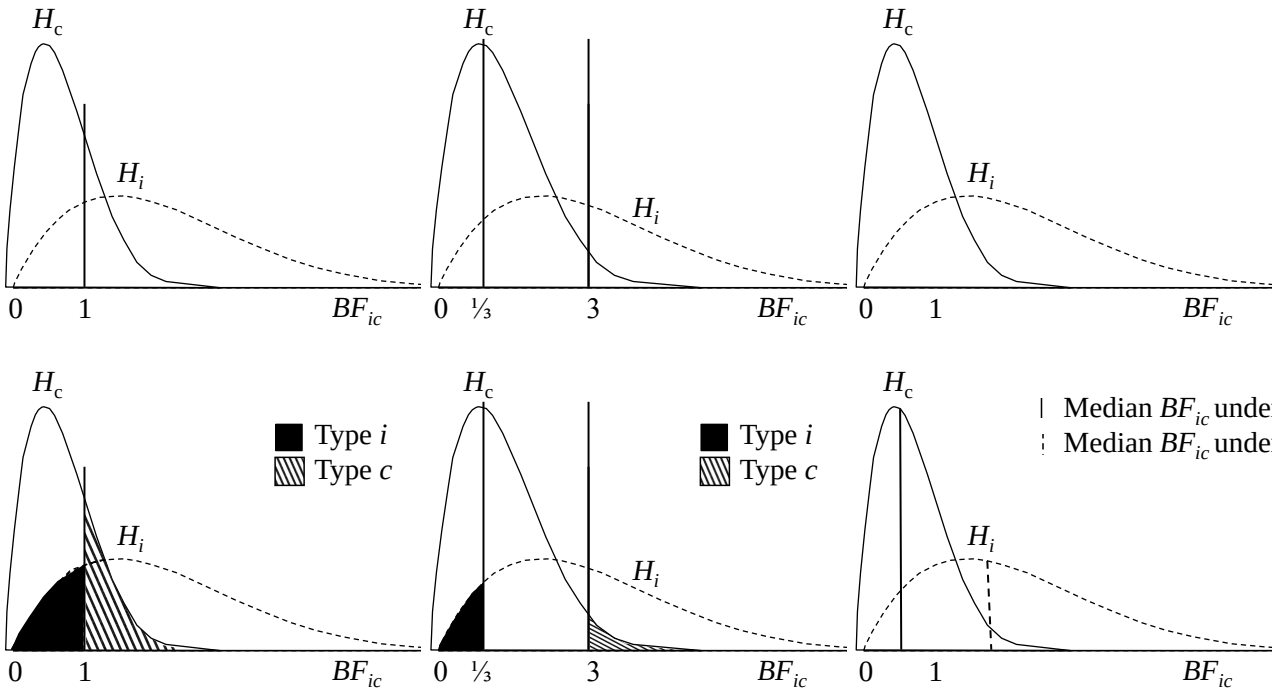
Researchers need to

- Formulate hypotheses (H_i, H_c, or $H_{i'}$)
- Choose effect sizes in both hypotheses (d)
- Select approach
  - Select decision criterion
  - Choose critical value

Researchers get

- Answers to specific questions
- Smaller required sample sizes

Discussion (1)



... ...

Discussion (2)

	Error probability		
	.1		
d_{H_i}	.2	.5	-
s	644	624	977
m	136	56	103
l	109	35	54
-	159	28	

Cohen: $K = 3,\ \beta = .2, \alpha = .05$

.2	.5
322	52

- H_i and H_c versus H_0 and H_1

Discussion (2)

	Error probability		
	.1		
d_{H_i}	.2	.5	-
s	644	624	977
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.2	.5
322	52

- H_i and H_c versus H_0 and H_1
- H_c
  - What does it mean?
  - What population to sample from?
  - What effect size to choose?
  - How to aggregate sample sizes over different parts of H_c?

Discussion (2)

	Error probability		
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d_{H_i}	.2	.5	-
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Cohen: $K = 3,\ \beta = .2, \alpha = .05$

.2	.5
322	52

- H_i and H_c versus H_0 and H_1
- H_c
  - What does it mean?
  - What population to sample from?
  - What effect size to choose?
  - How to aggregate sample sizes over different parts of H_c?
- H_i versus a composite informative hypothesis?
  - $H_j : \mu_1 > \mu_2 > \mu_3$ OR $\mu_2 > \mu_1 > \mu_3$

Thank you for listening

Effect size

	K	d
H_i	2, 3, 4	.2, .5
H_c	2, 3, 4	.2

Effect size

	K	d
H_i	2, 3, 4	.2, .5
H_c	2, 3, 4	.2

$$d = \frac{\mu_1 - \mu_K}{\sigma}$$

Effect size

	K	d
H_i	2, 3, 4	.2, .5
H_c	2, 3, 4	.2

$$d = \frac{\mu_1 - \mu_K}{\sigma}$$

K	d	μ_1	μ_2	μ_3
	0.2	0.2	0.1	0
3	0.5	0.5	0.25	0
	0.8	0.8	0.4	0

Results Approach 1: H_i vs $H_{i'}$

			Error probability								
			.1			.05			.025		
K		d_{H_i} / $d_{H_{i'}}$	.2	.5	-	.2	.5	-	.2	.5	-
		.2	318	147	318	531	327	531	731	531	731
	s	.5	147	51	51	327	88	88	531	117	117
		-	318	51		531	88		731	117	
		.2	103	54	108	180	108	180	252	180	249
3	m	.5	54	18	17	103	29	30	180	40	41
		-	103	18		180	29		252	40	
		.2	81	41	81	135	81	135	192	135	192
	l	.5	41	14	14	81	22	22	135	31	31
		-	81	14		135	22		192	31	

indicates controlled Type i error probability

indicates controlled Type i' error probability

indicates controlled Decision error probability

Cohen: $K = 3$, $\beta = .2$,$\alpha = .05$

small	medium
322	52

Results Approach 1: H_i vs $H_{i'}$

			Error probability								
			.1			.05			.025		
K		d_{H_i} / $d_{H_{i'}}$	.2	.5	-	.2	.5	-	.2	.5	-
		.2	318	147	318	531	327	531	731	531	731
	s	.5	147	51	51	327	88	88	531	117	117
		-	318	51		531	88		731	117	
		.2	103	54	108	180	108	180	252	180	249
3	m	.5	54	18	17	103	29	30	180	40	41
		-	103	18		180	29		252	40	
		.2	81	41	81	135	81	135	192	135	192
	l	.5	41	14	14	81	22	22	135	31	31
		-	81	14		135	22		192	31	

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small	medium
322	52

Results Approach 1: H_i vs $H_{i'}$

			Error probability								
			.1			.05			.025		
K		d_{H_i} / $d_{H_{i'}}$	.2	.5	-	.2	.5	-	.2	.5	-
		.2	318	147	318	531	327	531	731	531	731
	s	.5	147	51	51	327	88	88	531	117	117
		-	318	51		531	88		731	117	
		.2	103	54	108	180	108	180	252	180	249
3	m	.5	54	18	17	103	29	30	180	40	41
		-	103	18		180	29		252	40	
		.2	81	41	81	135	81	135	192	135	192
	l	.5	41	14	14	81	22	22	135	31	31
		-	81	14		135	22		192	31	

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small	medium
322	52

Results Approach 1: H_i vs $H_{i'}$

			Error probability								
			.1			.05			.025		
K		d_{H_i} / $d_{H_{i'}}$	.2	.5	-	.2	.5	-	.2	.5	-
		.2	318	147	318	531	327	531	731	531	731
	s	.5	147	51	51	327	88	88	531	117	117
		-	318	51		531	88		731	117	
		.2	103	54	108	180	108	180	252	180	249
3	m	.5	54	18	17	103	29	30	180	40	41
		-	103	18		180	29		252	40	
		.2	81	41	81	135	81	135	192	135	192
	l	.5	41	14	14	81	22	22	135	31	31
		-	81	14		135	22		192	31	

indicates controlled Type i error probability

indicates controlled Type i' error probability

indicates controlled Decision error probability

Cohen: $K = 3$, $\beta = .2$,$\alpha = .05$

small	medium
322	52

Approach	Critical value		Effect size		
	Type	Size	.2	.5	.8
Cohen	Decision error	.125	393	64	26
Approach 1	Decision error	.100	82	40	36
Approach 2	Decision error	.100	20	8	6
	Indecision		*.422*	*.384*	*.343*
Approach 2b	Indecision	.100	182	108	108
	Decision error		*.005*	*.007*	*.007*
Approach 3	B under H_i	10	87	15	6
	$P(BF_{ic} < 1 \mid H_i)$		*.091*	*.089*	*.091*
Approach 3	$1/B$ under H_c	$\frac{1}{10}$	87		
	$P(BF_{ic} > 1 \mid H_c)$		*.091*		

Note. Required sample sizes for the evaluation of H_i and H_c with $K = 2$. Effect size indicates d for Cohen (1992), and d_{H_i} for Approach 1–3. Note that $d_{H_c} = .2$ for all approaches. Note that Cohen's approach compares H_0 and H_1, while Approaches 1–3 compare H_i and H_c. Entries in italics are additional probabilities rendered by an approach.